where $A_P = (1/2)t_i L N_f$ is the profile area and $A_R = \sigma \varepsilon r_i^3 T_b^3/\kappa$ is a reference area. Therefore $(A_P/A_R)$ may be determined. The combination of $N_f$ and $N_L$ which gives the minimum value of $A_P/A_R$ is the optimum design.

Several analyses of this type were carried out for various values of $T_b$, $r_i$, and $Q$ to determine the ranges of parameters which might occur in optimally designed arrays. In all cases the optimum number of fins was found to be in the range $4 \leq N_f \leq 6$. Results are therefore given only for those values.

It should also be noted that the variation of heat transfer from the fin array with $\varepsilon$ was found to be nearly linear in the range $0.8 \leq \varepsilon \leq 1.0$. Linear interpolation is therefore valid in this range.

### Conclusions

Radiation from an array of longitudinal fins of triangular profile is analyzed including fin to fin and fin to base interactions. The effect of base cylinder radiation and the fin-base radiative interaction is found to be significant for $N_L$ less than 8. The results presented here may be used to optimize the design of a fin array with respect to weight.

### References

[1] Sparrow, E. M., Miller, G. B., and Jonsson, V. K., "Radiative Effectiveness of Annular Finned Space Radiators Including Mutual Irradiation Between Radiator Elements," *Journal of the Aerospace Sciences*, Vol. 29, 1962, pp. 1291–1299.

[2] Schnurr, N. M. and Cothran, C. A., "Radiation From an Array of Gray Circular Fins of Trapezoidal Profile," *AIAA Journal*, Vol. 12, Dec. 1974, pp. 1476–1480.

[3] Karlekar, B. V. and Chao, B. T., "Mass Minimization of Radiating Trapezoidal Fins with Negligible Base Cylinder Interaction," *International Journal of Heat Mass Transfer*, Vol. 6, 1963, pp. 33–48.

[4] Sparrow, E. M. and Cess, R. C., *Radiation Heat Transfer*, Brooks/Cole Publishing Co., Belmont, Calif., 1966, pp. 89–90.

# Reverse Transition at an Expansion Corner in Supersonic Flow

R. NARASIMHA* AND P. R. VISWANATH†
*Indian Institute of Science, Bangalore, India*

### Nomenclature

$c_{f_o}$ = $\tau_o/\frac{1}{2}\gamma p_o M_o^2$
$c_p$ = $(p_1 - p_o)/\frac{1}{2}\gamma p_o M_o^2 = -\Delta p/\frac{1}{2}\gamma p_o M_o^2$
$L$ = length of boat-tail surface
$M_o$ = freestream Mach number upstream of corner
$p_o$ = freestream static pressure upstream of corner
$p_1$ = freestream static pressure just downstream of corner
$R_o$ = Reynolds number per inch upstream of corner
$R_{\theta_o}$ = Reynolds number based on momentum thickness
$\tau_o$ = wall shear stress upstream of corner
$\delta_o$ = boundary-layer thickness upstream of corner
$\theta_o$ = momentum thickness upstream of corner
$\varepsilon$ = expansion (or corner) angle in deg
$\gamma$ = ratio of specific heats

### I. Introduction

MANY investigators[1–7] have reported direct or indirect evidence indicating that in supersonic flow past an expansion corner the boundary layer reverts from a turbulent state upstream of the corner to a laminar state downstream. The present work is an attempt to identify a criterion for the occurrence of such reversion, in particular for use in calculations of base pressure[8] on boat-tailed bodies. At low speeds, highly accelerated turbulent boundary-layer flows have received considerable attention in recent years,[9–11] and different criteria for the occurrence of reversion, each often implicitly adopting a different means of recognizing the phenomenon, have been suggested. In particular, the theoretical work of Narasimha and Sreenivasan,[11] which employs the ratio of the pressure gradient to a characteristic Reynolds stress gradient as the parameter governing the completion of reversion, has been successful in correlating and predicting the flow development, particularly in the later stages of reversion. The basic idea in this study, namely that during reversion the Reynolds stresses have little influence in large parts of the flow, should be valid with even greater force in supersonic flow past an expansive corner. Indeed the parameter mentioned, namely the quantity $p'\delta/\tau_o$ (where $p'$ is the pressure gradient, $\delta$ is the local boundary thickness, and $\tau_o$ is the wall shear stress in the boundary layer just upstream of the pressure gradient), takes on a simpler form in supersonic corner flow. For, the interaction of the expansion fan with the boundary layer spreads the pressure drop $\Delta p$ (here considered positive in an expansion) over a few $\delta$: the extent of this region appears to be insensitive to the total corner flow deflection, from Murthy and Hammitt's work.[4] Thus $-p' \sim \Delta p/\delta$, and consequently $-p'\delta/\tau_o \sim \Delta p/\tau_o$. This reasoning suggests that reversion will occur if $\Delta p/\tau_o$ is sufficiently large.

Received October 8, 1974.
Index categories: Boundary-Layer Stability and Transition; Supersonic and Hypersonic Flow.

* Professor, Department of Aeronautical Engineering.
† UGC Research Fellow, Department of Aeronautical Engineering.

### II. Data Analysis

To test this argument, we have examined all available experimental data (listed in Table 1 together with some relevant information), taking reversion to have occurred if there is evidence either of the growth of a thin new shear layer from the corner, or of a substantial change in the associated flow characteristics downstream (e.g., base pressure). This definition is appropriate to engineering calculations of recovery factor, skin friction, etc. Although it is not as precise as one might desire, nothing better is possible at present for supersonic flows, and the final results appear to justify it.

In the majority of the flows listed in Table 1 (the exception being Ref. 5), no value of $\tau_o$ is quoted, so we estimate it by the following procedure. First, it is assumed that the turbulent boundary layer upstream of the corner is fully developed and in equilibrium at constant pressure. Secondly, as all the experiments have been conducted under nearly adiabatic conditions, it is assumed that the skin friction coefficient just upstream of the corner, say $c_{f_o}$, depends only on the local Mach number $M_o$ and Reynolds number $R_{\theta_o}$ (based on freestream conditions and momentum thickness $\theta_o$). For this $M_o$ and $R_{\theta_o}$, $c_{f_o}$ is obtained from the theoretical results of Tetervin.[12] (Over the range of $M_o$ and $R_{\theta_o}$ covered in Table 1, a good fit to Tetervin's curves is given by

$$c_{f_o} = 0.0165\, M_o^{-0.36} R_{\theta_o}^{-0.20} \qquad (1)$$

which is therefore a convenient expression for estimating $\tau_o$.)

### III. Classification of Data

The flows listed in Table 1 are plotted in Figs. 1 and 2 after classification into three categories. a) Those in which the authors themselves reported indications of reversion (shown by filled symbols in Figs. 1 and 2); b) Those in which reversion (in the sense described in Sec. II) is inferred here (shown by filled, flagged symbols); and c) Those in which no evidence of reversion can be found (shown by open symbols).

Each flow listed in Table 1 has been scrutinized in the light of the above classification in Ref. 13; only a brief outline of this examination is given. Group a consists of the experiments of Sternberg,[1] Vivekanandan[2] and Viswanath and Narasimha.[3] In Sternberg's experiments, reversion was indicated by recovery factor measurements, which dropped from the turbulent value upstream of the corner to the laminar value downstream.

**Table 1 Data on flow past expansion corners**

| Author | $M_o$ | $R_o \times 10^{-6}$ | ε deg | Symbol | $c_{f_o}$ | $-c_p$ | $-c_p / c_{f_o}$ | Remarks |
|---|---|---|---|---|---|---|---|---|
| Sternberg[1] | 1.89 | 0.33 | 29 | S1 ▲ | 0.0036 | 0.340 | 92 | *a* |
| | | 0.46 | 29 | S2 ▲ | 0.0035 | 0.340 | 97 | *a* |
| | | 0.59 | 29 | S3 ▲ | 0.0033 | 0.340 | 103 | *a* |
| | | 0.73 | 29 | S4 ▲ | 0.00328 | 0.340 | 104 | *a* |
| | 2.15 | 0.26 | 29 | S5 ▼ | 0.0037 | 0.276 | 75 | *a* |
| | | 0.35 | 29 | S6 ▼ | 0.0035 | 0.276 | 79 | *a* |
| | | 0.59 | 29 | S7 ▼ | 0.0032 | 0.276 | 86 | *a* |
| Vivekanandan[2] | 2.02 | 0.90 | 25 | V1 ◆ | 0.0025 | 0.290 | 115 | *a* |
| Viswanath and Narasimha[3] | 1.36 | 0.78 | 4 | N1 ● | 0.0021 | 0.139 | 66 | *b* |
| | | | 10 | N2 ● | | 0.30 | 142 | *a* |
| Ananda Murthy and Hammitt[4] | 1.88 | 2.7 | 5 | A1 ▷ | 0.0018 | 0.098 | 54 | *c* |
| | | | 10 | A2 ◀ | | 0.177 | 98 | *b* |
| | | | 20 | A3 ◀ | | 0.287 | 160 | *b* |
| | | | 30 | A4 ◀ | | 0.347 | 192 | *b* |
| Morkovin[5] | 1.76 | 0.26 | 12 | M1 ◀ | 0.0021 | 0.224 | 106 | *b* |
| | | | | | 0.0014[d] | | 160 | *b* |
| Fuller and Reid[15] | 2.3 | 0.27 | 2.5 | F1 ▽ | 0.0027 | 0.0356 | 13 | *c* |
| | | | 5 | F2 ▽ | 0.0027 | 0.073 | 27 | *c* |
| | | | 7.5 | F3 ▽ | 0.0027 | 0.105 | 39 | *c* |
| Chapman et al.[14] | 1.495 | 0.57 | 5 | C1 □ | 0.0032 | 0.157 | 49 | *c* |
| | 1.50 | 0.56 | 8 | C2 ■ | 0.00294 | 0.210 | 71 | *b* |
| | 1.50 | 0.56 | 11 | C3 ■ | 0.00294 | 0.272 | 93 | *b* |
| | 1.50 | 0.56 | 14 | C4 ■ | 0.00294 | 0.328 | 111 | *b* |
| | 1.905 | 0.58 | 5.4 | C5 □ | 0.00276 | 0.103 | 37 | *c* |
| | 1.85 | 0.58 | 8.6 | C6 □ | 0.00286 | 0.159 | 55 | *c* |
| | 1.873 | 0.59 | 10.6 | C7 □ | 0.00278 | 0.188 | 67 | *c* |

[a] Reversion indicated by the authors. [b] Reversion inferred here. [c] No sign of reversion. [d] Value estimated using momentum integral technique.

Vivekanandan distinguished an inner or secondary layer near the wall from the rest of the boundary layer downstream of the corner, and found that the velocity profile in this inner layer was closer to that in laminar rather than turbulent flow; reversion was strongly suspected based on such observations. In the experiments of Viswanath and Narasimha on two-dimensional sharply boat-tailed bases, a reverse-transitional boundary layer along the boat-tail surface was strongly suspected from base pressure measurements as well as some qualitative hot-wire observations.

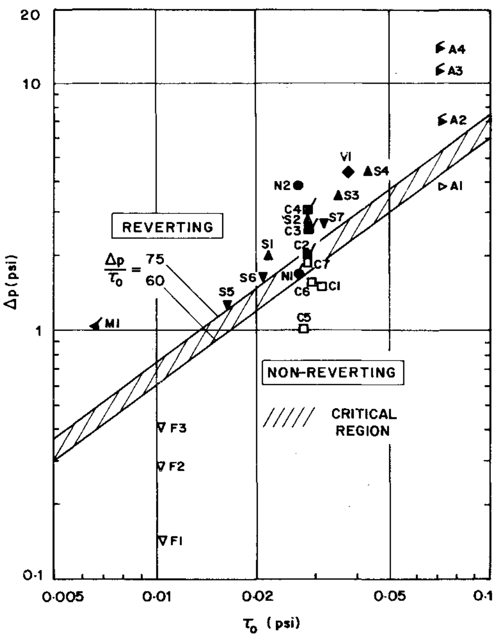


**Fig. 1 Pressure drop and wall stress in flow past expansion corner. (Filled symbols = experiments whose authors found indication of reversion; filled, flagged symbols = those where reversion is inferred here; open symbols = those where no sign of reversion can be found. For source of data, see Table 1.)**

Classification into Groups b and c requires a detailed consideration of each flow, as outlined. Ananda Murthy and Hammitt[4] do not draw any conclusion about possible reversion in any of their experiments, but schlieren photographs show the growth of a new shear layer from the corner, very clearly in the flows A3, A4 (as designated in Table 1), and less so in A2. Hence these are included in Group b. Flow A1 (with a 5° expansion), on the other hand, shows no such layer, and hence has been included in Group c.

Morkovin[5] experimented with the interaction between the turbulent boundary layer on the nozzle liner and an expansion fan generated away from the wall, but the interaction region was so small (about 2 $\delta_o$) that his study can be legitimately included in the present survey. He made no comment about possible reversion, but the observed decreases in turbulence intensity are consistent with measurements in low speed reverting flows.[9] We have, therefore, classified this flow M1 also in Group b.

Flows N1, C2, C3, and C4, all on boat-tailed two-dimensional bodies involving a sharp expansive corner, are judged to belong

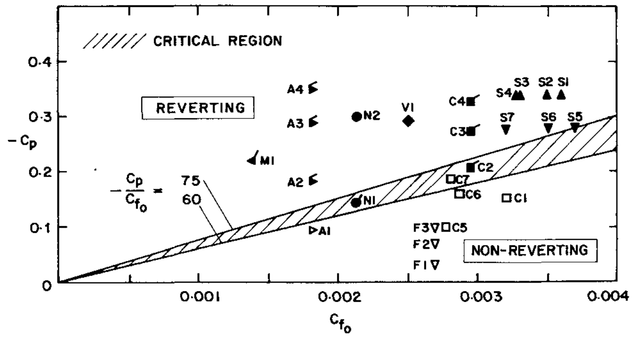


**Fig. 2 Pressure and skin friction coefficients in flow past expansion corner. Symbols as in Fig. 1.**

to Group b on the basis of base pressure measurements. These show base pressures substantially different from what would be expected[8] if the boundary layer at the separation point were fully turbulent. On the other hand, flows F1, F2, F3, C1, C5, C6, and C7 are classed in Group c, because the measured base pressures are in agreement with the turbulent correlation.[8] It must be noted that if the boat-tail length $L$ (i.e., distance from expansion corner to base) is sufficiently large, a boundary layer that underwent severe distortion at the corner may recover (e.g., by retransition to turbulence) before it reaches the base. In Sternberg's experiments, retransition occurred at a distance of about 20 $\delta_o$ from the corner, but there are not enough data on boat-tailed bases to judge how much $L$ should be for boundary-layer distortion effects to disappear. Presumably this will depend on Reynolds number and a variety of other factors. It is possible that the absence of a noticeable effect on base pressure in C6 and C7 ($\Delta p/\tau_o \simeq 50$ to 70, $L/\delta_o \gtrsim 7$) is partly due to the length of the boat-tail surface, but no definite conclusions are warranted at this stage.

### IV. Conclusion

It is interesting to see from Fig. 1 that all points above the line $\Delta p/\tau_o = 75$ belong to Groups a and b,‡ and all those below the line $\Delta p/\tau_o = 60$ belong to Group c. Between these two lines are a few points from both Groups b and c. Considering the uncertainties in estimating $\tau_o$, there is a remarkable consistency among the experimental data in indicating that the parameter $\Delta p/\tau_o$ governs the occurrence of reversion, which is likely whenever $\Delta p$ is more than about 70 $\tau_o$.

In Figs. 1 and 2, we have not plotted the experimental results (of reverting flows) on cooled blunted cone-cylinder models reported in Ref. 7, because of the difficulty of estimating $\tau_o$. Using the curves of Tetervin at the values of $M_o$, $R_{\theta_o}$ and temperature ratio quoted in Ref. 7, however, we obtain $\Delta p/\tau_o \simeq 50$ in cases (i) and (ii) of Ref. 7 in which laminar solutions for heat transfer agreed with the measurements downstream of the corner, and $\Delta p/\tau_o \simeq 3$ in case (iii), where the turbulent heat transfer solution was closer to the measurements. Considering the likely errors in estimating $c_{f_o}$ on cooled walls, it appears that the parameter $\Delta p/\tau_o$ can be used to predict the occurrence of reversion by expansion on cooled surfaces also.

Figure 2 shows the same data in terms of $c_p$ and $c_{f_o}$. (Note that $\Delta p/\tau_o = -c_p/c_{f_o}$.) Now $c_{f_o}$ varies only by a factor of about 2 between all the different experiments, whereas $\tau_o$ varies by a factor of 7 as shown in Fig. 1. If the variation of $c_{f_o}$ can be ignored, we may use the crude thumb rule that the flow may be expected to revert if $-c_p \gtrsim 0.2$. Estimating $c_p$ by supersonic linear theory, this rule takes the form $\varepsilon \gtrsim 5.74\,(M_o^2-1)^{1/2}$ deg for reversion, and should be useful for Mach numbers less than about 3 and $R_{\theta_o}$ less than about $10^5$. Its validity outside this range remains to be tested.

### References


[1] Sternberg, J., "The Transition from a Turbulent to a Laminar Boundary Layer," Rept. 906, May 1954, Ballistic Research Labs., Aberdeen, Md.

[2] Vivekanandan, R., "A Study of Boundary Layer Expansion Fan Interactions near a Sharp Corner in Supersonic Flow," 1963, M.Sc. thesis, Dept. of Aeronautical Engineering, Indian Institute of Science, Bangalore, India.

[3] Viswanath, P. R. and Narasimha, R., "Base Pressure on Sharply Boat-Tailed Aft Bodies," Rept. 72 FM1, July 1972, Dept. of Aeronautical Engineering, Indian Institute of Science, Bangalore, India.

[4] Ananda Murthy, K. R. and Hammitt, A. G., "Investigation of the Interaction of a Turbulent Boundary Layer with Prandtl-Meyer Expansion Fan at $M = 1.88$," Rept. 434, 1958, Dept. of Aeronautical Engineering, Princeton University, Princeton, N.J.

[5] Morkovin, M. V., "Effects of High Acceleration on a Turbulent Supersonic Shear Layer," *Proceedings, Heat Transfer and Fluid Mechanics Institute*, Los Angeles, Calif., June 1955.

[6] Page, R. H. and Sernas, P., "Apparent reverse transition in an expansion fan," *AIAA Journal*, Vol. 8, Jan. 1970, pp. 189–190.

[7] Zakkay, V., Toba, K., and Kuo, T.-J., "Laminar, Transitional, and Turbulent Heat Transfer after a Sharp Convex Corner," *AIAA Journal*, Vol. 2, Aug. 1964, pp. 1389–1395.

[8] Viswanath, P. R. and Narasimha, R., "Two-Dimensional Boat-Tailed Bases in Supersonic Flow," *Aeronautical Quarterly*, Vol. 25, Aug. 1974, pp. 210–224.

[9] Badri-Narayan, M. A. and Ramjee, V., "Criteria for Reverse Transition in a Two-Dimensional Boundary Layer Flow," Journal of Fluid Mechanics, Vol. 35, Feb. 1969, pp. 225–241.

[10] Patel, V. C. and Head, M. R., "Reversion of Turbulent to Laminar Flows," *Journal of Fluid Mechanics*, Vol. 34, Nov. 1968, pp. 371–392.

[11] Narasimha, R. and Sreenivasan, K. R., "Relaminarization in Highly Accelerated Turbulent Boundary Layers," *Journal of Fluid Mechanics*, Vol. 61, Nov. 1973, pp. 417–447.

[12] Tetervin, N., "An Analytical Investigation of the Flat Plate Turbulent Boundary Layer in Compressible Flow," NOLTR 67-39, Aerodynamics Research Rept. 286, May 1967, Naval Ordnance Lab., Silver Spring, Md.

[13] Narasimha, R., and Viswanath, P. R., "Reverse Transition at an Expansion Corner in Supersonic Flow," Rept. 74 FM 10, Aug. 1974, Dept. of Aeronautical Engineering, Indian Institute of Science, Bangalore, India.

[14] Chapman, D. R., Wimbrow, W. R., and Kester, R. H., "Experimental Investigation of Base Pressure on Blunt Trailing Edge Wings at Supersonic Velocities," Rept. 1109, 1952, NACA.

[15] Fuller, L. and Reid, J., "Experiments in Two-Dimensional Base Flow at $M = 2.4$," ARC R & M 3064, Aeronautical Research Council, London, 1958.


‡ Morkovin's flow has $\Delta p/\tau_o = 160$ using the value of $c_{f_o}$ quoted by him. This value (obtained from momentum integral balance) is rather low for the $M_o$, $R_{\theta_o}$ of the experiment, as Morkovin himself notes. Equation (1) gives a value about 50% higher, but even this gives $\Delta p/\tau_o \simeq 100$, well above the critical region of Fig. 1.

# The Auxiliary Problem for Feasible Directions


L. B. GWIN* AND G. N. VANDERPLAATS†

*NASA Ames Research Center, Moffett Field, Calif.*


### Introduction

FEASIBLE direction methods provide an effective means of obtaining solutions to nonlinear, constrained optimization problems. Of these methods, one of the most widely used is that of Ref. 1. Zoutendijk showed that a sequence of improvements to the objective function can be generated by the careful selection of a direction vector $\bar{S}$ at constraint boundaries, followed by a one-dimensional search in this direction until the next boundary is reached. The determination of the $\bar{S}$ vector is a subproblem which is usually solved for structural applications as a linear programing problem in an $n$-dimensional design space using the simplex algorithm. An alternative approach formulates the subproblem with a single quadratic constraint, which in turn can be cast as a special linear problem and solved directly. This Note compares the computational efficiency of these two means of determining the $\bar{S}$ vector and demonstrates the advantages of using the alternative approach.

### Problem Statement

The direction vector is computed as the solution to the following auxiliary extremum problem, for which $\beta$ is maximized subject to the conditions[1–3]


Received October 25, 1974; revision received November 4, 1974.

Index categories: Computer Technology and Computer Simulation Techniques; Structural Design, Optimal.



* NRC Research Associate. Member AIAA.

† Research Scientist. Member AIAA.